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Rheological Relation for Optical Materials in Dynamic Photoelasticity

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Abstract: **The method of calculating the difference principal stresses and strains in linearly viscoelastic optically sensitive polymer materials by the given picture of interference fringes, which is changing in time, is offered. The main value obtained using direct and inverse Laplace transforms.**

Keywords: **photoelasticity, solid mechanics, stress, strain, polymers.**

I. INTRODUCTION

The photoelastic method stands out among the methods used in solid mechanics [1]. The method is characterized by the possibility of identifying the distribution of mechanical stress and strains from a fringe pattern without the need to process experimental data. In the study of stress in solids by the method of dynamic photoelasticity [2], it is necessary to know the viscoelastic properties of optically sensitive model of polymers that are isotropic with respect to the elastic properties.

Research of mechanical stress by isotropic-photoelastic method for modeling the mechanical behavior associated with the solution of following methodological problems; the choice of material model, similar in its mechanical behavior to the materials of natural body, determination of stresses and strains in the model; transition to similar mechanical quantities in nature.

II. STATE OF THE PROBLEM AND MAIN EQUATIONS

Usually when determining stress, they use linear dependence of the band order m only on the strip through the optical stress coefficient C_δ, or only through deformation by optical strain coefficient C_ε:

$$m = C_\delta(\sigma_1 - \sigma_2); \qquad m = C_\varepsilon(\varepsilon_1 - \varepsilon_2) \tag{1}$$

However, the known equation Filon-Jessop, simultaneously included stress and strain [4] defined by:

$$m = C_\delta^*(\sigma_1 - \sigma_2) + C_\varepsilon^*(\varepsilon_1 - \varepsilon_2) \tag{2}$$

If the relation between stresses and strains are known, then constant coefficients in (1) and (2) are linked. Note that the dynamic nature of the load in viscoelastic materials coefficients C_σ and C_ε depend on strain rate.

In the study of the behavior of linear viscoelastic properties piezo optic materials, Mindlin [3] used the model that contained four elements of solid deformable body. He has shown that for incompressible material for proportional changing load coefficients C_σ and C_ε replaced by operators which depend on time. These polarization directions coincide with each other as well as deformation and stress.

Note that in general the main axis of stresses may not coincide with the axes of polarization under dynamic loading. The discrepancy between the axes can take a relatively small part of the total time of observation for quasi-static problems, i.e only for viscoelastic transition. So, for engineering calculactions acceptable hypothesis of convergence of optical and mechanical axes of deformation over all time could be considered.

Mechanical behavior of elastic isotropic homogeneous body can be described by two independent constants, which are bulk compression modulus K and shear modulus G. Denote deviator components of stress σ_{ij} and strain ε_{ij} by ζ_{ij}, in e_{ij} respectively, we can write:

$$\xi_{ij} = 2Ge_{ij}; \qquad \sigma_{ij} = 3K\varepsilon_{ij} \tag{3}$$

For linear viscoelastic body isothermal relationship stress-strain conveniently interpreted through linear differential operators []

$$\left[a_n \frac{\partial^n}{\partial t^n} + ... + a_0\right]\xi_{ij}(x_k,t) = \left[b_m \frac{\partial^m}{\partial t^m} + ... + b_0\right]e_{ij}(x_k,t) \tag{4}$$

$$\left[c_r \frac{\partial^r}{\partial t^r} + ... + c_0\right]\sigma_{ij}(x_k,t) = \left[d_s \frac{\partial^s}{\partial t^s} + ... + d_0\right]e_{ij}(x_k,t) \tag{5}$$

Where a_n, b_m, c_r and d_s are determined experimentally.

We apply the Laplace transform to equations (4) and (5) and zero initial conditions we get:

$$\xi_{ij}(x_{k,}p) = \frac{[b_m p^m + ... + b_0]}{[a_n p^n + ... + a_0]}\bar{e}_{ij}(x_{k,}p) \equiv 2G(p)\bar{e}_{ij}(x_{k,}p); \tag{6}$$

$$\bar{\varepsilon}_{ij}(x_{k,}p) = \frac{[d_s p^s + ... + d_0]}{[c_r p^r + ... + c_0]}\varepsilon_{ii}(x_{k,}p) \equiv 3K(p)\varepsilon_{ii}(x_{k,}p) \tag{7}$$

From equations (6) and (7) it follows that relationship between stresses and strains has pseudo-elastic character in the plane transformation. To characterize the material it is necessary to define operators G_p in K_p. If these operators are known, then other characteristics could be obtained from the formulae:

$$E(p) = \frac{9G(p)K(p)}{3K(p) + 6G(p)} \tag{8}$$

$$\gamma(p) = \frac{3K(p) - 2G(p)}{6K(p) + 2G(p)} \tag{9}$$

where E is Young's moduli , and v is Poisson's ratio.

Consider the three main aspects of the approximation behavior of the material:

a) not compressible material in terms of volume deformation and viscoelastic behavior with shear;

b) Poisson's ratio is constant, but differs from 0.5 and is consistent with the data of [Mindlin], which showed that Poisson's ratio for epoxy polymer ED -16 Ma under pulsed load equal 0.35;

c) Material is linear isotropic and homogeneous

For the first case:

$$v(p)=0,5; \quad K(p)=\infty; \quad E(p)=3G(p) \tag{10}$$

For the second case:

$$v(p)=v_0; \qquad K(p) = \frac{2G(p)(1+\nu_0)}{3(1-2\nu_0)}; \qquad E(p)=2(1+v_0)G(p) \tag{11}$$

For the third case:

$$K(p)=\frac{2G(p)[1+\nu(p)]}{3[1-2\nu(p)]}\ ;\ E(p)=2[1+\nu(p)]G(p) \qquad (12)$$

The inverse Laplace transform for (12) is associated with considerable mathematical difficulties.

The first case is considered in detail in Williams work [7] so focus on the second case.

In the main stresses and strains equation (4) has the form

$$\frac{\bar{\sigma}_1(p)-\bar{\sigma}_2(p)}{2}=G(p)[\bar{\varepsilon}_1(p)-\bar{\varepsilon}_2(p)] \qquad (13)$$

For axial tensile take: $\sigma_1=\sigma;\ \ \sigma_2=0;\ \varepsilon_1=\varepsilon;\ \ \varepsilon_2=-\nu_0\,\varepsilon$. Then (13) is written:

$$\bar{\sigma}(p)=2G(p)\bar{\varepsilon}(p)-(1+\nu_0)=E(p)\bar{\varepsilon}(p) \qquad (14)$$

where $E(p)=2(1+\nu_0)G(p)$.

If stress relaxation $\varepsilon(t)=\varepsilon_0$ at $t>0$, then $\bar{\varepsilon}(p)=\frac{\varepsilon_0}{P}$.

By definition module relaxation:

$$E_{pe\Lambda}(t)=\frac{\sigma_{pe\Lambda}(t)}{\varepsilon_0}\ ;\ \bar{E}_{pe\Lambda}(p)=\frac{\bar{\sigma}_{pe\Lambda}(p)}{\varepsilon_0} \qquad (15)$$

Equation (13) in experiments on stress relaxation can be written as:

$$\sigma_{pe\Lambda}(p)=E(p)\frac{\varepsilon_0}{p}$$

Due to this, we can write (15) as:

$$E(p)=p\bar{E}_{pe\Lambda}(p);\ \ G(p)=p\bar{G}_{pe\Lambda}(p) \qquad (16)$$

Similarly to (4) and (5) the relationship between the optical and mechanical values can be written using linear differential operators, replacing constant coefficient C_σ and C_ε by corresponding operators:

$$\sigma_1-\sigma_2=C_\delta^{-1}[m] \qquad (17)$$

$$\varepsilon_1-\varepsilon_2=C_\varepsilon^{-1}[m] \qquad (18)$$

Applying the Laplace transform to equations (17) and (18) we obtain:

$$\bar{m}(x_i,p)=C_\delta(p)[\bar{\sigma}_1(x_i,p)-\bar{\sigma}_2(x_i,p)] \qquad (19)$$

$$\bar{m}(x_i,p)=C_s(p)[\bar{\varepsilon}_1(x_i,p)-\bar{\varepsilon}_2(x_i,p)] \qquad (20)$$

Apply (13) we get:

$$C_s(p)=2G(p)C_\delta(p) \qquad (21)$$

Consider the relationship (18), which is linking the relative optical path difference with the main difference deformations for different load cases.

In experiments on relaxation :

$$C_{\varepsilon_{pe\Lambda}}(t)=\frac{m_{pe\Lambda}(t)}{\varepsilon_1-\varepsilon_2} \qquad (22)$$

Since $\varepsilon_1=\varepsilon_0$, $\varepsilon_2=-\nu\varepsilon_0$, then

$$\bar{C}\varepsilon_{pe\Lambda}(t)=\frac{\bar{m}_{pe\Lambda}(t)}{\varepsilon(1+\nu_0)} \qquad (23)$$

Knowing variable time of the picture intervention bands, we calculate the difference principal stress, making use of its superposition integral:

$$\sigma_1(t)-\sigma_2(t)=m(0)C^{-1}_{\sigma_{pe\Lambda}}(t)+\int_0^t C^{-1}_{\sigma_{pe\Lambda}}(t-\tau)\frac{\partial m(\tau)}{\partial\tau}\partial\tau \qquad (24)$$

Accordingly, the main difference equation for deformation will look like:

$$\varepsilon_1(t)-\varepsilon_2(t)=m(0)C^{-1}_{\varepsilon}(t)+\int_0^t C^{-1}_{\varepsilon}(t-\tau)\frac{\partial m(\tau)}{\partial\tau}\partial\tau \qquad (25)$$

So, to find the main difference stresses and strains need to know

$C^{-1}_{\sigma_{pe\Lambda}}(t)$ i $C^{-1}_{\varepsilon}(t)$.

Using the result obtained in (5) the relations can be written in form

$$\bar{C}^{-1}_{\varepsilon_{pe\Lambda}}(p)=2p\bar{G}_{pe\Lambda}(p)\bar{C}_{\sigma_{pe\Lambda}}(p)$$

$$\bar{C}_{\sigma_{pe\Lambda}}(p)=\frac{1}{p^2\bar{C}_{\sigma_{noB}}(p)}$$

Find the relation between $C^{-1}_{\sigma_{pe\Lambda}}(t)$ and $C^{-1}_{\varepsilon}(t)$, $\bar{C}_{\varepsilon_{pe\Lambda}}(p)$:

$$\bar{C}^{-1}_{\sigma_{pe\Lambda}}(p)=\frac{2G_{pe\Lambda}(p)}{p\bar{C}\varepsilon_{pe\Lambda}(p)} \qquad (26)$$

$$\bar{C}^{-1}_{\varepsilon_{noB}}(p)=\frac{1}{p^2\bar{C}_{\varepsilon_{pe\Lambda}}(p)} \qquad (27)$$

We apply inverse Laplace transform to these relations, we can find $C^{-1}_{\sigma_{pe\Lambda}}(t)$ and $C^{-1}_{\varepsilon}(t)$.

In problems of dynamic analysis of the stress state, when it is necessary to know the properties at very small values of time (10^{-5} – 10^{-3} c)) applicable of the principle of temperature-time analogy. In accordance with this principle, depending on the type of mechanical parameter *P* is logarithm of time *lgt* does not change with temperature, only shifting the curve along the axis *lgt* on the value of a_T, which determines how many times the speed decreases relaxation process at temperature *T*, according to its speed at a certain temperature T_0

According to (5), the relation for coefficient a_T has the form:

$$\lg a_T(T)=-\frac{K_1(T/T_0)}{K_2+(T-T_0)} \qquad (28)$$

III. CONCLUSION

Thus, analysis of research results at different temperatures and strain rate, and also use of the relation (28) make it possible to construct a complete dynamic spectrum. Having determined in experiments for relaxation at low temperatures $G^{-1}_{\varepsilon_{pe\Lambda}}(t)$ and using temperature-temporal analogy and formulas (26) and (27), we can find $\bar{C}^{-1}_{\sigma_{pe\Lambda}}(p)$ and $\bar{C}^{-1}_{\varepsilon_{no\mathrm{B}}}(p)$ at small intervals of the load. Using the inverse Laplace transform we obtain $C^{-1}_{\sigma_{pe\Lambda}}(t)$ and $C^{-1}_{\varepsilon}(t)$ and use the formulas (24) and (25), we can find the stress and strain in the studied model.

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